Registration No.:

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E.(Full Time) - END SEMESTER EXAMINATIONS, APR/MAY 2024

Second Semester

MA7251 MATHEMATICS II

(Regulations 2015)

Time: 3 Hours

Maximum Marks: 100

Answer ALL Questions

Part-A (10x2=20 Marks)

1. If the eigen values of a 3×3 square matrix A are -1, 1 and 2, what is the determinant of A?
2. If the eigen values of a 3×3 matrix are 1, 2 and 3, write down the eigen values of the inverse of the matrix.
3. If ϕ is a scalar field, define $grad(\phi)$.
4. What is the directional derivative of the function $\phi(x, y, z) = x + y + z$ at (1,2,3) in the direction of the vector $i + j + k$?
5. Is the function $f(z) = e^z$ conformal? Justify your answer.
6. Let $f(z) = \overline{z}$ (conjugate of z). Is $f(z)$ analytic? Justify your answer.
7. What is an essential singularity?
8. State Cauchy's integral theorem.
9. Find $L[t\sin(t)]$.
10. Find $L[(t-1)^2]$.

Part-B (5x16=80 Marks)

11 (i) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(8 Marks)

(ii) Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$$

(8 Marks)

12 (a) Verify Gauss divergence theorem for

$$\overline{F} = (x^2 - yz)i + (y^2 - xz)j + (z^2 - xy)k$$

taken over the cuboid $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$. (16 Marks)

(OR)

12 (b) (i) Show that the gradient of a nice scalar field is always irrotational and that curl of a well-behaved vector field is always solenoidal. (8 Marks)

(ii) Prove that

$$\overline{F} = (x^2 - yz)i + (y^2 - xz)j + (z^2 - xy)k$$

is conservative and also find its scalar potential (8 Marks)

13 (a) (i) Find the analytic function $f(z) = u + iv$ if $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. (8 Marks)

(ii) If $f(z) = u + iv$ is analytic in a region, prove that u and v are harmonic in the region. (8 Marks)

(OR)

13 (b) (i) Find the bilinear transformation that maps the points 1, 0, i onto the points 1, ∞, $-i$. Also, find the fixed points of such a transformation. (8 Marks)

(ii) Find the image of $2 < x < 3$ under the mapping $w = 1/z$. (8 Marks)

14 (a) (i) Expand the following function in the region $1 < |z| < 3$:

$$f(z) = \frac{1}{(z+1)(z+3)}$$

(8 Marks)

(ii) Evaluate the following integral

$$\int_C \frac{4-3z}{z(z-1)(z-2)} dz$$

where C is the circle $|z| = 3/2$, using Cauch's integral formula. (8 Marks)

(OR)

14 (b) Using contour integration, evaluate the following:

$$\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$$

(16 Marks)

15 (a) (i) Use Laplace transform to solve $y'' - 4y' + 3y = e^{-t}$, $y(0) = 0$ and $y'(0) = 0$. (8 Marks)

(ii) Find the Laplace transform of periodic function. (8 Marks)

(OR)

15 (b) (i) Using convolution theorem, find

$$L^{-1}\left[\frac{s^2}{(s^2+1)^2}\right]$$

(8 Marks)

(ii) If $f(t)$ is periodic with period 3 and $g(t)$ is periodic with period 4, are the sum $f(t) + g(t)$ and the product $f(t)g(t)$ periodic? Justify your answer. (8 Marks)